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QUESTION BANK

(FOR STUDENTS ENROLLED IN THE YEAR 2014-15 & LATER)

Class: **F.Y.B.Sc.** Subject: **MATHEMATICS** Paper**: II** Sem: **II**

Title of Paper: **LINEAR ALGEBRA**

UNIT I: SYSTEM OF LINEAR EQUATIONS AND MATRICES:

DEFINITIONS:

1. Linear equation (in n unknowns)
2. System of Linear equations
3. Homogeneous System of Linear Equations
4. Non-Homogeneous System of Linear Equations
5. Product of Matrices
6. Scalar Matrix
7. Diagonal Matrix
8. Upper Triangular Matrix
9. Lower Triangular Matrix
10. Symmetric Matrix
11. Skew- Symmetric Matrix
12. Invertible Matrix
13. Transpose of a Matrix
14. Zero Row
15. Non-Zero Row
16. Pivot(leading coefficient)
17. Row Echelon Form of a Matrix
18. Solution of a linear system
19. Trivial solution of a system
20. Non-trivial solution of a system
21. Row - equivalent matrices

PROBLEMS:

TYPE I (Converting to row echelon form):

Convert the following matrices into row echelon matrices:

(a) $\begin{bmatrix} 1 & -3 & 3 \\ 4 & 7 & 12 \\ 2 & 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 & -7 & 7 \\ 3 & 1 & -3 & 5 \\ 4 & 2 & -8 & 20 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & -3 \\ 3 & 5 & 1 \\ 12 & 20 & 4 \\ 9 & 15 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & -5 \\ 4 & 8 & 10 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & 4 \\ 0 & 0 \\ 14 & 10 \end{bmatrix}$.

TYPE II (Solving systems using Gaussian Elimination Method):

Solve the following system of linear equations. Also, give the geometric interpretation of the solution sets.

(a) $x - 2y + 3z = 1;$ $3x + y - 2z = 5;$ $5x - 3y + 4z = 7.$

(b) $2x + 3y - z = 0;$ $x + y + z = 0.$

(c) $5x + 3y + 7z - 4 = 0;$ $3x + 26y + 2z - 9 = 0;$ $7x + 2y + 10z - 5 = 0.$

(d) $x + 2y - z = 3;$ $3x - y + 2z = 1;$ $2x - 2y + 3z = 2;$ $x - y + z = -1.$

(e) $x + 7y - 2 = 0;$ $3x + 21y - 1 = 0.$

(f) $x + y + z = 3;$ $x + 2y + 2z = 5;$ $3x + 4y + 4z = 12.$

TYPE III(Finding Parametric Equations):

1. Find parametric equations of line passing through the following points:
   (a) $(2,-4),(-3,-1)$ (b) $(1,-1,0),(2,0,-7)$
   (c) $\left(3,0,-\frac{7}{3}\right),(0,\frac{1}{2},-1)$ (d) $(1,-3),(-2,6)$.

2. Find the parametric equations of plane passing through the following points:
   (a) $(1,-1,0),(2,0,-7),(0,0,-1)$ (b) $(1,2,3),(2,4,6),(0,0,1)$
   (c) $(1,0,0),(0,1,0),(0,0,1)$ (d) $(1,1,1),(0,1,1),(0,0,1)$.

PROPOSITIONS:

1. If A is an $m \times p$ matrix and B is a $p \times n$ matrix then prove that $(AB)^T = B^T A^T$.
2. If A and B are invertible matrices then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
3. Prove that matrix multiplication is associative. i.e., if A is an $m \times l$ matrix, B is an $l \times p$ matrix and C is a $p \times n$ matrix then $(AB)C = A(BC)$.
4. Prove that any system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.
5. Prove that any matrix is row equivalent to a matrix in row echelon form.
6. Prove that for every square matrix A over $\mathbb{R}$, $A + A^t$ is a symmetric matrix and $A - A^t$ is a skew-symmetric matrix.
7. Prove that if the determinant of the coefficient matrix of a homogeneous system of two linear equations in two unknowns is non-zero then the system has only trivial solution.
8. Prove that a necessary and sufficient condition for the sum of two solutions or a scalar multiple of a solution to be a solution of the same system of linear equations is that the system is homogeneous.

MISCELLANEOUS:

1. Give the geometric interpretation of all possible solution sets of m linear equations in n unknowns when (i) $m = 1,\ n = 2$, (ii) $m = 2, n = 2$, (iii) $m = 2, n = 3$, (iv) $m = 3, n = 2$, (v) $m = n = 3$.
2. If A and B are $n \times n$ symmetric matrices over $\mathbb{R}$ then prove that $A + B$ and αA are symmetric for every $\alpha \in \mathbb{R}$.
3. Under what condition would a diagonal matrix, a scalar matrix and an upper triangular matrix be invertible?
4. For the system $x - y = 3, 2x + 3y = 4, 3x + 2y = k$, using Gaussian elimination method, find the real value k such that the given system has a solution. Hence, solve the given system.
5. Let A be an $n \times n$ matrix such that A^3 is zero matrix. Prove that $I - A$ is invertible where I is the $n \times n$ identity matrix.
6. Using parametric equations, check whether the points $(1,2,4),(0,-1,2)$ and $(-1,-4,0)$ are collinear.
7. Let P be the plane passing through the point $A(1,2,3)$ and perpendicular to the vector $n = (0,2,-1)$. Let l be the line passing through $B(2,4,1)$ in the direction of n. Find the point of intersection of plane P and line l.
8. If A and B are $n \times n$ matrices over $\mathbb{R}$ such that $AB = I$ where I is $n \times n$ identity matrix then prove that $BA = I$.
9. Let $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\forall \theta \in \mathbb{R}$. Prove that $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)\ \ \forall \theta_1, \theta_2 \in \mathbb{R}$.
10. Using parametric equations of line and plane, prove that the distance of a point $P(x_1, y_1, z_1)$ from a plane $ax + by + cz + d = 0$ is $\frac{|\,ax_1+by_1+cz_1+d\,|}{\sqrt{a^2+b^2+c^2}}$.

UNIT II: VECTOR SPACES:

DEFINITIONS:

1. Vector Space over$\mathbb{R}$.
2. Vector Subspace
3. Linear Combination of vectors
4. Linear Span of a non-empty subset of a Vector Space
5. Generating Set of a Vector Space
6. Linear Independent Subset of a Vector Space
7. Linear Dependent Subset of a Vector Space.

PROBLEMS:

TYPE I(Proving the given spaces as Vector spaces and subspaces)

1. Proving that the following are Vector spaces and Vector Subspaces over $\mathbb{R}$:

Vector Space	Vector Subspace(Non Trivial)
$\mathbb{R}^n$ (alsofor n = 2,3 in particular)	1. Any Line passing through the origin 2. Any Plane passing through the origin 3. Space of solutions of m homogeneous linear equations in n unknowns.
$\mathbb{R}[x]$	$P_n[x]$
$M_{m\times n}(\mathbb{R})$	1. Space of all upper triangular matrices $(form = n = 2,3)$ 2. Space of all lower triangular matrices $(form = n = 2,3)$ 3. Space of all diagonal matrices $(form = n = 2,3)$ 4. Space of all symmetric matrices $(form = n = 2,3)$ 5. Space of all skew-symmetric matrices $(form = n = 2,3)$
$F(X,\mathbb{R})$ where $X \neq \phi$	$C(X,\mathbb{R})$ ≔Space of all continuous real valued functions defined on X

2. Check whether the following are Vector Spaces over $\mathbb{R}$:
1. $V = \{(x_1,x_2,x_3)\epsilon\mathbb{R}^3 | x_1,x_2,x_3$ are rationals $\}$ wrt usual addition and scalar multiplication as in $\mathbb{R}^3$.
2. $V = \{(x_1,x_2)\epsilon\mathbb{R}^2 | x_1 = 2x_2 + x_3\}$ wrt usual addition and scalar multiplication as in$\mathbb{R}^2$.
3. $V =$ Set of all real sequences with first term equal to 1.
i.e., $V = \{(x_n) | (x_n)$ is a real sequence with x_1 =1$\}$ wrt componentwise addition and scalar multiplication.
4. $V =$ Set of all real sequences with first term equal to 0.
i.e., $V = \{(x_n) | (x_n)$ is a real sequence with x_1 =0$\}$ wrt componentwise addition and scalar multiplication.

3. Check whether the following are vector subspaces of the given Vector Spaces over $\mathbb{R}$:
1. $W = \{(x,y,z)\ \epsilon\mathbb{R}^3 | z = x + y + 1\}$ of $\mathbb{R}^3$
2. $W = \{(x,0,z)\epsilon\mathbb{R}^3 | x,z\ \epsilon\mathbb{R}\}$ of $\mathbb{R}^3$
3. $W = \{(x_1,x_2,x_3)\epsilon\mathbb{R}^3 | x_1 + x_3 \neq 0\}$ of $\mathbb{R}^3$.
4. $W = \{f: X \to \mathbb{R} | f(x_0) = 0\}$ of $F(X,\mathbb{R})$.(Note that x_0 is fixed in $X \neq \phi$).
5. $W = \{A\epsilon M_2(\mathbb{R}) | AB = BA\}$ of $M_2(\mathbb{R})$. (Note that B is a fixed matrix in $M_2(\mathbb{R})$).
6. $W = \{(x,y,z) \in \mathbb{R}^3 | z = 2x - y\}$ of $\mathbb{R}^3$.

4. Give an example to show that $W_1 \cup W_2$ may not be a vector subspace of a vector space V even if W_1 and W_2 are subspaces of V.

TYPE II (Expressing a vector as a linear combination)

1. Express the vector v as a linear combination of the other given vectors:
1. $v = (1,3), v_1 = (1,2), v_2 = (-1,2)$
2. $v = (5,3,-1), v_1 = (2,0,0), v_2 = (0,1,-1), v_3 = (1,0,2)$
3. $v = 1 + 2t + 3t^2$, $v_1 = 1$, $v_2 = 1 + t, v_3 = 1 + t^2$
4. $v = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$

2. Check whether
   1. (1,2,3) can be written as a linear combination of (1,0,1) and (1,1,1).
   2. (1,2,4,5) can be written as a linear combination of the vectors (1,0,0,0), (0,1,0,0) and (0,1,1,0).

<u>TYPE III</u>(Problems involving Linear Spans, Generating Sets)

1. Find the linear span of the given subsets of the given Vector Spaces over $\mathbb{R}$:
   1. $S = \{(1,0), (-2,1)\}$ of $\mathbb{R}^2$
   2. $S = \{(1,0,0,0), (0,1,1,0)\}$ of $\mathbb{R}^4$
   3. $S = \{\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}\}$ of $M_2(\mathbb{R})$
   4. $S = \{1, 1+2x, 1-2x+x^2+x^3, 2x^3\}$ of $P_3[x]$.
2. Show that $S = \{(1,1), (-1,3)\}$ generates $\mathbb{R}^2$.
3. Check whether {(1,0,0), (0,1,1), (2,3,1)} generates $\mathbb{R}^3$.
4. Write down a subset of $M_2(\mathbb{R})$ that generates $M_2(\mathbb{R})$.
5. Find the subspace of $\mathbb{R}^3$generated by {(1,0,0), (1,1,0), (1,0,1)}.

<u>TYPE IV</u>(Problems involving Linear Independence/Dependence)

1. Determine whether the following are linearly dependent/independent in the given vector spaces:
   1. {(1,1), (-1,1), (0,3) } in $\mathbb{R}^2$.
   2. { (2,0,0) } in $\mathbb{R}^3$.
   3. $\{1-x, x(1-x), 1-x^2\}$ in $P_2[x]$.
   4. $\{(-1,1,1), (2,1,1), (1,2,2)\}$ in $\mathbb{R}^3$.
   5. $\{\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}, \begin{bmatrix}0 & 2\\5 & 0\end{bmatrix}, \begin{bmatrix}1 & 3\\10 & 2\end{bmatrix}\}$ in $M_2(\mathbb{R})$.
   6. $\{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}, \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\}$ in $M_2(\mathbb{R})$.

<u>PROPOSITIONS</u>

1. Let V be a vector space over $\mathbb{R}$ and W be a non empty subset of V. Prove that W is a vector subspace of V iff $\alpha.w_1 + \beta.w_2 \epsilon$ W $\forall \alpha, \beta \in \mathbb{R}$ and $\forall w_1, w_2 \epsilon$ W.
2. Prove that the set of all solutions of a homogeneous system of m linear equations in n unknowns is a vector subspace of $\mathbb{R}^n$.
3. Let W_1 and W_2 be vector subspaces of a vector space V over $\mathbb{R}$. Prove that $W_1 \cap W_2$ is a vector subspace of V.
4. Prove that arbitrary intersection of vector subspaces of a vector space is a vector subspace.
5. Let W_1 and W_2 be vector subspaces of a vector space V over $\mathbb{R}$. Prove that $W_1 \cup W_2$ is a subspace of V iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.
6. Let $S(\neq \emptyset) \subseteq V$, a vector space over $\mathbb{R}$. Prove that the Linear Span of S $i.e., L(S)$ is a vector subspace of V.
7. Prove that a set of vectors in a vector space is linearly dependent iff atleast one of the vectors in the set is a linear combination of the other vectors.

<u>MISCELLANEOUS</u>:

1. Show that the set of all polynomials over $\mathbb{R}$ of degree equal to 5 does not form a vector space over $\mathbb{R}$ under usual addition and scalar multiplication.
2. Show that every subset of a finite linearly independent subset of a vector space over $\mathbb{R}$ is linearly independent.
3. Show that every superset of a finite linearly dependent subset of a vector space over $\mathbb{R}$ is linearly dependent.
4. Let $S = \{v_1, v_2, \dots, v_n\}$ be a linearly independent subset of a vector space over $\mathbb{R}$. Prove that for any vector v, $S \cup \{v\}$ is linearly dependent iff $v \in L(S)$.

UNIT III: BASIS AND LINEAR TRANSFORMATION:

DEFINITIONS:

1. Basis of a vector space
2. Finitely generated vector space
3. Maximal linearly independent set
4. Minimal generating set
5. Dimension of a vector space
6. Sum of vector subspaces
7. Linear Transformation
8. Kernel of a linear transformation
9. Image of a linear transformation
10. Nullity of a linear transformation
11. Rank of a linear transformation
12. Matrix of a linear transformation

STATEMENT:

Rank-Nullity Theorem: Let V be a finitely generated vector space over $\mathbb{R}$ and W be any vector space over $\mathbb{R}$. If $T: V \to W$ is a linear transformation then Rank(T) + Nullity(T) = dim(V)

PROBLEMS:

TYPE I (Checking for Basis):

Check whether the following sets form a basis for the given vector spaces:

i. $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$

ii. $\{(1,0,0),(1,1,0),(0,1,1)\}$ for $\mathbb{R}^3$

iii. $\{1, 1+t, 1+t^2\}$ for $P_2[t]$

iv. $\{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\}$ for $M_2(\mathbb{R})$

v. $\{(1,0,0),(1,1,0),(5,-1,0)\}$ for $\mathbb{R}^3$

vi. $\{(1,0),(1,1),(-1,2)\}$ for $\mathbb{R}^2$

TYPE II(Sum of vector subspaces):

Find the dimension of $W_1 + W_2$ where

i. $W_1 = x$ - axis, $W_2 = y$ - axis

ii. $W_1 = x$ - axis, $W_2 = xy$ plane

iii. $W_1 = xy$ plane, $W_2 = yz$ plane

iv. $W_1 = xy$ plane, $W_2 = xz$ plane

v. $W_1 = \{0\}$, $W_2 = \mathbb{R}^2$

vi. $W_1 = \{(x,x) |\ x \in \mathbb{R}\}$, $W_2 = x$ – axis

TYPE II (Checking for Linear Transformations):

Check whether following are linear transformations:

i) $T: \mathbb{R} \to \mathbb{R}$ defined as $T(x) = 2x$

ii) $T: \mathbb{R} \to \mathbb{R}$ defined as $T(x) = 2x+1$

iii) $T: \mathbb{R} \to \mathbb{R}$ defined as $T(x) = x^2$

iv) $T: \mathbb{R}^2 \to \mathbb{R}^4$ defined as $T(x,y) = (x+y, 2x, 2y, x-y)$

v) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x,y) = (x+2y, 2x, 2)$

vi) $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x,y) = (x-y, |x|)$

vii) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x,y) = (2x+y, 2x, 0)$

viii) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x,y,z) = (x-3y, x^2, 2y+z)$

ix) $T: C[0,1] \to \mathbb{R}$ defined as $T(f) = \int_0^1 f$

x) $T: C[0,1] \to \mathbb{R}$ defined as $T(f) = f(t_0)$ for some fixed t_0 in $[0,1]$

xi) $T: P_5[t] \to P_5[t]$ defined as $T(f) = \frac{df}{dt}$
xii) $T: M_2[\mathbb{R}] \to M_2[\mathbb{R}]$ defined as $T(A) = 2A$

<u>TYPE III</u> (Finding Kernels):

Find the Kernels for the following Linear Transformations:
i) $T: \mathbb{R}^2 \to \mathbb{R}^4$ defined as $T(x,y) = (x+y, 2x, 2y, x-y)$.
ii) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x,y) = (2x+y, 2x, 0)$.
iii) $T: P_5[t] \to P_5[t]$ defined as $T(f) = \frac{df}{dt}$.
iv) $T: M_2[\mathbb{R}] \to M_2[\mathbb{R}]$ defined as $T(A) = 2A$.
v) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x,y,z) = (2x+y, x-y+z, 3x+z)$.
vi) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x,y,z) = (2x+y, 2x, 0)$.
vii) $T: \mathbb{R}^3 \to \mathbb{R}$ defined as $T(x,y,z) = 0$.

<u>TYPE IV</u>(Finding Image spaces):
Find the Image space for the following Linear Transformations:
i) $T: \mathbb{R}^2 \to \mathbb{R}^4$ defined as $T(x,y) = (x+y, x, 0\ y)$.
ii) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x,y) = (2x+y, 0, 0)$.
iii) $T: P_5[t] \to P_5[t]$ defined as $T(f) = \frac{df}{dt}$.
iv) $T: M_2[\mathbb{R}] \to M_2[\mathbb{R}]$ defined as $T(A) = 2A$.
v) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x,y,z) = (2x+y, x-y+z, 3x+z)$.
vi) $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x,y,z) = (2x+y, 2x, 0)$.
vii) $T: \mathbb{R}^3 \to \mathbb{R}$ defined as $T(x,y,z) = 0$.

<u>TYPE V</u>(Verifying Rank – Nullity Theorem):
Verify the rank-nullity theorem for the following linear transformations:

i. $T: \mathbb{R}^3 \to \mathbb{R}$ defined as $T(x,y,z) = 0$.
ii. $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined as $T(x,y,z) = (x,y)$.
iii. $T: \mathbb{R}^3 \to \mathbb{R}$ defined as $T(x,y,z) = x$.
iv. $T: \mathbb{R}^3 \to \mathbb{R}$ defined as $T(x,y,z) = y$.
v. $T: P_2[t] \to P_2[t]$ defined as $T(f) = \frac{df}{dt}$.

<u>TYPE VI</u> (Finding Matrices associated to Linear Transformations):

i) Find matrix with respect to standard bases for the linear transformations given in TYPE III - i, ii, v, vi, vii.
ii) Find matrix for the linear transformation given in TYPE III - iii with respect to basis $\{1, t, t^2, t^3, t^4, t^5\}$.
iii) Find matrix for the linear transformation given by TYPE III - iv with respect to basis $\left\{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\right\}$.

<u>PROPOSITIONS:</u>

1. Prove that every basis of a finitely generated vector space is maximal linearly independent.
2. Prove that every maximal linearly independent subset of a vector space is a basis of the vector space.
3. Prove that every basis of a finitely generated vector space is minimal generating.
4. Prove that every minimal generating subset of a finitely generated vector space is a basis of the vector space.
5. Prove that any set of $n+1$ vectors in a vector space with n elements in a basis is linearly dependent.

6. Prove that any two bases of a vector space have the same number of elements.
7. Prove that any n linearly independent vectors in an n- dimensional vector space forms a basis of the vector space.
8. Prove that if W_1 and W_2 are vector subspaces of a vector space V over $\mathbb{R}$ then $W_1 + W_2$ is a vector subspace of V.
9. Prove that if W_1 and W_2 are vector subspaces of a vector space V over $\mathbb{R}$ then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.
10. Let V and W be vector spaces. Let $T: V \to W$ be a linear transformation. Prove that $Ker\ T$ is a subspace of V and $Im\ T$ is a subspace of W.
11. Let V and W be vector spaces. Let $\{v_1, v_2, \dots, v_n\}$ be a basis of V. Let $w_1, w_2, \dots . w_n$be any arbitrary vectors of W. Prove that there exits a unique linear transformation $T: V \to W$ such that $T(v_i) = w_i$ for $i = 1,2, \dots, n$.

MISCELLANEOUS:

1. Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1,0) = (2,3)$ and $T(0,1) = (3,2)$.
2. Find the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,0) = (2,3)$ and $T(0,1,0) = (3,2)$ and $T(0,0,1) = (1,2)$.
3. If $T: \mathbb{R}^2 \to \mathbb{R}$ is a linear transformation such that $T(0,1) = (1,2)$ and $T(1,0) = (1,4)$ then find $T(2,3)$.
4. Find a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that the matrix of T wrt the standard bases is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
5. Prove that any linear transformation maps zero vector to zero vector.
6. Find the rank of the linear transformation $T: P_2[t] \to P_2[t]$ defined as $T(f) = \frac{df}{dt}$ using the rank-nullity theorem.
7. What is the dimension of the zero vector space i.e., V={0}? Justify.
8. Give a basis of the vector space $\mathbb{R}$ over $\mathbb{R}$. Hence, write dimension of $\mathbb{R}$ over $\mathbb{R}$.
